Oxford Revise | Edexcel A Level Maths | Answers

- Method (**M**) marks are awarded for showing you know a method and have attempted to apply it.
- Accuracy (**A**) marks should only be awarded if the relevant M marks have been awarded.
- Unconditional accuracy (**B**) marks are awarded independently of M marks. They do not rely on method.
- The abbreviation **o.e.** means 'or equivalent (and appropriate)'.

Please note that:

- efficient use of advanced calculators is expected
- inexact numerical answers should be given to three significant figures unless the question states otherwise; values from statistical tables should be quoted in full
- when a value of g is required, it is taken as $g = 9.8\ \text{m s}^{-2}$ unless stated otherwise in the question.

Chapter 20 Gradients and graphs

Question	Answer	Extra information	Marks
20.1 (a)	$f'(x) = 6x + 2$	Attempting to differentiate	M1
		Correct derivative	A1
	$f'(2) = 14$	Evaluating correctly	A1
20.1 (b)	$y = 15$	y-value at $x = 2$	B1
	$y - 15 = 14(x - 2)$	Use of any correct formula	M1
	$y = 14x - 13$	Correct equation	A1

Question	Answer	Extra information	Marks
20.1 (c)	$m_N = -\frac{1}{14}$ $y - 15 = -\frac{1}{14}(x - 2)$ $y = -\frac{1}{14}x + \frac{106}{7}$ (o.e.)	Correct gradient Use of any correct formula with their m_N Correct equation	B1 M1 A1
20.1 (d)	$x = -\frac{1}{3}$	Correct answer	B1
20.1 (e)	$3\left(-\frac{1}{3}\right)^2 + 2\left(-\frac{1}{3}\right) - 1 = -\frac{4}{3}$ Hence coordinates are $\left(-\frac{1}{3}, -\frac{4}{3}\right)$	Correct coordinates	B1
20.1 (f)	$f''(x) = 6$	Correct answer	B1
20.1 (g)	$f''(x)$ is positive so the turning point is a minimum.	Correct answer	B1
20.1 (h)	$f''(x) \geq 0$ for all x so the curve is convex.	Correct answer	B1
	Total		**14 marks**
20.2 (a)	$f'(x) = 3x^2 - 4x + 4$ $f'(1) = 3 - 4 + 4$ $= 3$	Attempting to differentiate Correct derivative Correct gradient	M1 A1 A1

Question	Answer	Extra information	Marks
20.2 (b)	$y = 0$ $y - 0 = 3(x - 1)$ $y = 3x - 3$	Identifying y-value at $x = 1$ Use of any correct formula Correct equation	B1 M1 A1
20.2 (c)	$x^3 - 2x^2 + 4x - 3 = 3x - 3$ $x^3 - 2x^2 + x = 0$ $x(x-1)^2 = 0$ Hence $x = 0$ or 1 Hence x-coordinate at $P = 0$ and $y = -3$ $P(0, -3)$	Equating two expressions Attempting to solve cubic Correct x-values Correct coordinates	M1 M1 A1 A1
20.2 (d)	$f''(x) = 6x - 4 \Rightarrow f''(x) = 0$ when $x = \frac{2}{3}$ $f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) - 3$ $= -\frac{25}{27}$ Point of inflection is at $\left(\frac{2}{3}, -\frac{25}{27}\right)$ $f''\left(\frac{19}{30}\right) = -\frac{1}{5}$ and $f''\left(\frac{21}{30}\right) = \frac{1}{5}$ so change of sign; therefore, it is a point of inflection.	Attempting to find f″ Correct solution of $f''(x) = 0$ Substituting their x into f Correct coordinates Any valid explanation	M1 A1 M1 A1 A1

Question	Answer	Extra information	Marks
20.2 (e)	$f'\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) + 4$ $= \frac{8}{3} \neq 0$ Hence the point of inflection is not stationary.	Substituting their x into f′ Correct conclusion	M1 A1
	Total		**17 marks**
20.3 (a)	$f'(x) = 1 - 4x^{-2}$ $f'\left(\frac{1}{2}\right) = -15$ $m_N = \frac{1}{15}$ $y = \frac{17}{2}$ $y - \frac{17}{2} = \frac{1}{15}\left(x - \frac{1}{2}\right)$ $y = \frac{1}{15}x + \frac{127}{15}$ (o.e.)	Differentiating Correct gradient of curve Correct normal gradient Identifying y-value for $x = \frac{1}{2}$ Use of any correct formula Correct equation	M1A1 A1 A1 B1 M1 A1

Question	Answer	Extra information	Marks
20.3 (b)	$1 - 4x^{-2} = 0$ $x = 2$ $f(2) = 2 + 2 = 4$ Hence turning point is at (2, 4) $f''(x) = 8x^{-3}$ $f''(2) = 1$ (> 0) so it is a minimum point.	Attempting to solve $f'(x) = 0$ Correct x-coordinate Correct y-coordinate Attempting to find f'' Correct conclusion	M1 A1 A1 M1 A1
20.3 (c)	$x \geq 2$	Correct answer	B1
20.3 (d)	$f''(x)$ is positive for all $x > 0$	Correct conclusion	B1
	Total		**14 marks**
20.4	$f'(x) = 6x^2 - 4$ At A: $f'(1) = 2$ and $f(1) = 3$ $y - 3 = 2(x - 1)$ $y = 2x + 1$ At B: $f'(4) = 92$ and $f(4) = 117$ $y - 117 = 92(x - 4)$ $y = 92x - 251$ C has x-coordinate where $2x + 1 = 92x - 251 \Rightarrow x = \frac{14}{5}$ When $x = \frac{14}{5}$, $y = \frac{33}{5}$, so the coordinates of C are $\left(\frac{14}{5}, \frac{33}{5}\right)$	Attempting to differentiate Correct derivative Correct coordinates for A Use of any correct formula Correct equation Correct coordinates for B Correct equation Equating lines and attempting to solve Correct coordinates for C	M1 A1 A1 M1 A1 A1 A1 M1 A1

Question	Answer	Extra information	Marks
	Total		**9 marks**
20.5 (a)	$f'(x) = 1 + \cos x$ Hence $f'\left(\frac{\pi}{3}\right) = \frac{3}{2}$	Attempting to differentiate Correct derivative Correct gradient	M1 A1 A1
20.5 (b)	$y = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ $y - \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) = \frac{3}{2}\left(x - \frac{\pi}{3}\right)$ $y = \frac{3}{2}x - \frac{\pi}{6} + \frac{\sqrt{3}}{2}$ o.e.	Identifying y-value at $x = \frac{\pi}{3}$ Use of any correct formula Correct equation	B1 M1 A1
20.5 (c)	$f''(x) = -\sin x$ $-\sin \pi = 0$ and hence $f''(x) = 0$ $-\sin 0.9\pi = -0.309\ldots$ $-\sin 1.1\pi = 0.309\ldots$ Since the second derivative changes sign either side of the point, it is a point of inflection.	Correct second derivative Evaluating $f''(\pi)$ Correct explanation	B1 B1 B1
	Total		**9 marks**
20.6 (a)	$f'(x) = 2 - 3e^{3x}$ $f''(x) = -9e^{3x}$ Since e^{3x} is positive for all x, $f''(x)$ is negative for all x, so $f(x)$ is concave for all x	Attempting to differentiate twice Correct second derivative Correct explanation	M1 A1 A1

Question	Answer	Extra information	Marks
20.6 (b)	$2 = 3e^{3x}$ $x = \frac{1}{3}\ln\left(\frac{2}{3}\right)$ o.e.	Attempting to solve f′(x) = 0 Correct x-coordinate. Must be exact.	M1 A1
20.6 (c)	$x \geq \frac{1}{3}\ln\left(\frac{2}{3}\right)$	Correct answer	B1
	Total		**6 marks**
20.7 (a)	$f(x) = 3x - 1 + 2x^{-1}$ $f'(x) = 3 - 2x^{-2}$ Hence $3 = 2x^{-2} \Rightarrow x = \sqrt{\frac{2}{3}}$ $y = 3\sqrt{\frac{2}{3}} - 1 + 2\sqrt{\frac{3}{2}}$ So the coordinates of the turning point are $\left(\sqrt{\frac{2}{3}},\ 3\sqrt{\frac{2}{3}} - 1 + 2\sqrt{\frac{3}{2}}\right)$	Attempting to write in index form and differentiating Correct derivative Attempting to solve f′(x) = 0 Both coordinates correct. Must be exact.	M1 A1 M1 A1
20.7 (b)	$f''(x) = 4x^{-3}$ When $x = \sqrt{\frac{2}{3}}$, $f''(x)$ is positive so turning point is a minimum.	Correct second derivative Correct explanation	A1 B1
20.7 (c)	$f''(x) = \frac{4}{x^3}$ which can never take a zero value.	Correct explanation	B1

Question	Answer	Extra information	Marks
	Total		**7 marks**
20.8 (a)	$f''(x) = \frac{1}{x} + \frac{3}{2}x^{\frac{1}{2}}$ For all $x > 0$, $f'(x)$ is positive so the function is increasing.	Attempting to differentiate Correct derivative Correct conclusion	M1 A1 A1
20.8 (b)	$f''(x) = -\frac{1}{x^2} + \frac{3}{4}x^{-\frac{1}{2}}$ Hence $\frac{1}{x^2} = \frac{3}{4}x^{-\frac{1}{2}} \Rightarrow \frac{4}{3} = x^{\frac{3}{2}}$ $x = \sqrt[3]{\frac{16}{9}}$ (= 1.211…) $f''(1.2) = -0.00979\ldots$ $f''(1.22) = 0.00715\ldots$ Since the second derivative changes sign, the point is a point of inflection.	Attempting to find f'' Correct second derivative Attempting to solve $f''(x) = 0$ Correct x Correct strategy to show change of sign Correct conclusion	M1 A1 M1 A1 M1 A1
20.8 (c)	$f'\left(\sqrt[3]{\frac{16}{9}}\right) = 2.47\ldots$ $f'(x) \neq 0$ so it is not a stationary point of inflection.	Must show that $f'(x) \neq 0$	B1
	Total		**10 marks**

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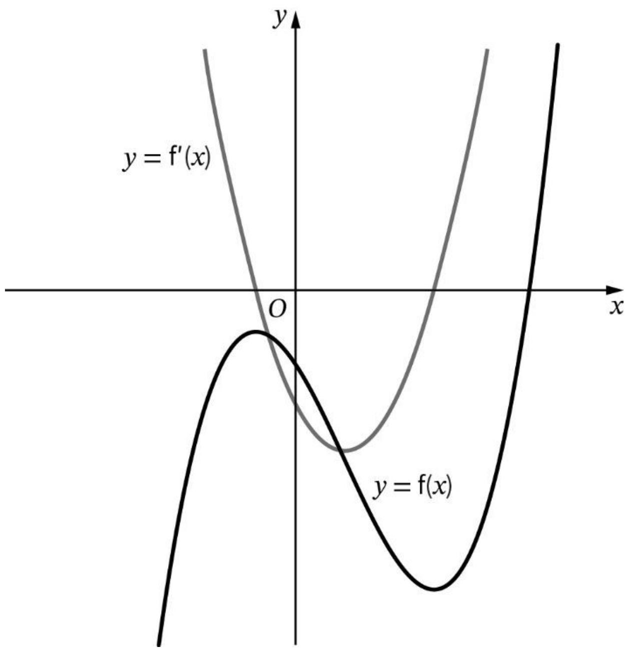
Question	Answer	Extra information	Marks
20.9 (a)	$f'(x) = \frac{1}{x} - 2x^{-2}$ $f'(1) = -1$ $y = 2$ $y - 2 = -(x - 1)$ so $y = -x + 3$	Attempting to differentiate Gradient at $x = 1$ Identifying y-coordinate for $x = 1$ Use of any correct formula Correct equation	M1 A1 A1 M1 A1
20.9 (b)	$\frac{1}{x} = 2x^{-2} \Rightarrow x = 2$ $y = \ln 2 + 1$	Attempting to solve $f'(x) = 0$ Correct x-coordinate Correct y-coordinate. Must be exact.	M1 A1 A1
20.9 (c)	$f''(x) = -\frac{1}{x^2} + 4x^{-3}$ When $x = 2$, $f''(x) = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$ which is positive, so the point is a minimum.	Attempting to find second derivative Substituting their x into f'' Correct conclusion	M1 M1 A1
	Total		**11 marks**

Question	Answer	Extra information	Marks
20.10	$f'(x) = 4x^3 - 12x^2 + 16$	Attempting to differentiate	M1
		Correct derivative	A1
	Solving $4x^3 - 12x^2 + 16 = 0$ gives $x = -1$ or $x = 2$	Correct solutions for $f'(x) = 0$	A1
	At $x = -1$, $y = -10$		
	At $x = 2$, $y = 17$	Both points identified	A1
	$f''(x) = 12x^2 - 24x$	Attempting to find f''	M1
	At $x = -1$, $f''(x) = 36$ (minimum point)	Identifying minimum	A1
	At $x = 2$, $f''(x) = 0$	Correct strategy to show change of sign	M1
	At $x = 1.9$, $f''(x) = -2.28$		
	At $x = 2.1$, $f''(x) = 2.52$		
	Change of sign means (stationary) point of inflection.	Identifying point of inflection	A1
	Total		**8 marks**

Question	Answer	Extra information	Marks
20.11		Root at maximum Root at minimum Turning point at point of inflection. Must be below *x*-axis. Overall shape is quadratic	B1 B1 B1 B1
	Total		**4 marks**
20.12 (a)	$30\pi = \pi r^2 h \Rightarrow h = \dfrac{30}{r^2}$ $A = 2\pi r \times \dfrac{30}{r^2} + \pi r^2$ $= 60\pi r^{-1} + \pi r^2$	Finding an expression for *h* Substituting their *h* into correct area formula Correct conclusion	B1 M1 A1

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Question	Answer	Extra information	Marks
20.12 (b)	$\frac{dA}{dr} = -\frac{60\pi}{r^2} + 2\pi r$ $\frac{60\pi}{r^2} = 2\pi r$ $r = \sqrt[3]{30}$ $= 3.107...$ $= 3.11$ (3 s.f.) $A = 91.0 \text{ cm}^2$	Attempting to find derivative Attempting to solve $\frac{dA}{dr} = 0$ Correct r Substitute their r into surface area formula. Can be implied from correct answer. Correct area	M1 M1 A1 M1 A1
20.12 (c)	$\frac{d^2A}{dr^2} = \frac{120\pi}{r^3} + 2\pi$ When $r = 3.11$, second derivative = 18.849… This is positive, so it is a maximum point.	Attempting to find second derivative and substituting their r into it Correct justification	M1 A1
	Total		**10 marks**

Question	Answer	Extra information	Marks
20.13	$u_{205} = 4 + 204 \times 3$ $= 616$ $\frac{n}{2}[8+(n-1)\times 3] < 616$ $\Rightarrow 3n^2 + 5n - 1232 < 0$ $n = 19.448\ldots$ (ignore negative solution) So $n = 19$	Attempting to find u_{205} Correct value Use of correct formula Identifying correct quadratic Correct n	M1 A1 M1 A1 A1
	Total		**5 marks**
20.14	$3x - 20° = \cos^{-1}(-0.35)$ $= 110.487\ldots$ and 249.512…, 470.487… $x = 43.5°, 89.8°, 163.5°$	Use of inverse cosine Principal solution Use of $360° \pm \alpha$ Identifying all required values All values of x correct	M1 A1 M1 A1 A1
	Total		**5 marks**