Oxford Revise | Edexcel A Level Maths | Answers

- Method (**M**) marks are awarded for showing you know a method and have attempted to apply it.
- Accuracy (**A**) marks should only be awarded if the relevant M marks have been awarded.
- Unconditional accuracy (**B**) marks are awarded independently of M marks. They do not rely on method.
- The abbreviation **o.e.** means 'or equivalent (and appropriate)'.

Please note that:

- efficient Use advanced calculators is expected
- inexact numerical answers should be given to three significant figures unless the question states otherwise; values from statistical tables should be quoted in full
- when a value of g is required, it is taken as $g = 9.8\ \text{m s}^{-2}$ unless stated otherwise in the question.

Chapter 24 Differential equations

Question	Answer	Extra information	Marks
24.1 (a)	$\int(3x^2+2x)\mathrm{d}x = x^3+x^2+c$	Attempting to integrate Must include '+ c'	M1 A1
24.1 (b)	$3 = 1^3 + 1^2 + c \Rightarrow c = 1$ Hence $y = x^3 + x^2 + 1$	Using the point in an equation involving c Correct answer	M1 A1
	Total		**4 marks**
24.2 (a)	$\int e^{2x}\mathrm{d}x = \frac{1}{2}e^{2x}+c$	Attempting to integrate Must include '+ c'	M1 A1

Question	Answer	Extra information	Marks
24.2 (b)	$4 = \frac{1}{2}e^0 + c \Rightarrow c = \frac{7}{2}$	Using the point in an equation involving c	M1
	Hence $8 = \frac{1}{2}e^{2x} + \frac{7}{2} \Rightarrow e^{2x} = 9$	Setting up and solving equation given their c	M1
	$x = \ln 3$, $y = 8$	Correct answer	A1
	Total		**5 marks**
24.3	$\int \cos 3x\,dx = \frac{1}{3}\sin 3x + c$	Attempting to integrate. Must include '+ c'.	M1A1
	$3 = \frac{1}{3}\sin\left(\frac{3\pi}{2}\right) + c \Rightarrow c = \frac{10}{3}$	Substituting boundary condition	M1
	Hence $y = \frac{1}{3}\sin 3x + \frac{10}{3}$	Correct equation	A1
	Total		**4 marks**

Question	Answer	Extra information	Marks
24.4	$\int \tan y \mathrm{d}y = \int \frac{x}{x^2-1}\mathrm{d}x$	Attempting to separate the variables Correct equation	M1 A1
	$\ln\lvert\sec y\rvert = \frac{1}{2}\ln\lvert x^2-1\rvert + c$	Attempting to integrate Must include '+c'	M1 A1
	$\ln\lvert\sec y\rvert = \ln\left\lvert(x^2-1)^{\frac{1}{2}}\right\rvert + c$	Using rule of logs	M1
	$\sec y = \mathrm{e}^c \mathrm{e}^{\ln\left\lvert(x^2-1)^{\frac{1}{2}}\right\rvert}$	Exponentials of both sides	M1
	$\sec y = \mathrm{e}^c (x^2-1)^{\frac{1}{2}}$	Removing logs	M1
	$\sec y = k\sqrt{x^2-1}$ as required	Complete correct derivation	A1
	Total		**8 marks**
24.5 (a)	$\int \frac{1}{y}\mathrm{d}y = \int (x^2+x)\mathrm{d}x$	Attempting to separate the variables	M1
	$\ln y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + c$	Attempting to integrate	M1
	$y = \mathrm{e}^{\frac{1}{3}x^3+\frac{1}{2}x^2+c}$	Attempting to rearrange	M1
	$y = A\mathrm{e}^{\frac{1}{3}x^3+\frac{1}{2}x^2}$	Correct equation	A1

Question	Answer	Extra information	Marks
24.5 (b)	$1 = Ae^{\frac{1}{3}\times 0^3 + \frac{1}{2}\times 0^2} = A$	Constant of integration	B1
	Hence $y = e^{\frac{1}{3}x^3 + \frac{1}{2}x^2}$		
	$\frac{dy}{dx} = 0 \Rightarrow x = -1$ or 0	Solving derivative equal to 0	M1
	Hence stationary points at (0, 1) and $\left(-1, e^{\frac{1}{6}}\right)$	Correct answer	A1
	Total		**7 marks**

Question	Answer	Extra information	Marks
24.6 (a)	$2x^2 + 5x + 2 = (2x + 1)(x + 2)$	Attempting to factorise and finding partial fractions	M1
	$\frac{-6}{(2x+1)(x+2)} = \frac{A}{x+2} + \frac{B}{2x+1}$		
	$-6 = A(2x + 1) + B(x + 2)$		
	$\frac{-6}{(2x+1)(x+2)} = \frac{2}{x+2} - \frac{4}{2x+1}$	Correct partial fractions	A1
	$\int \frac{1}{y-1}\,\mathrm{d}y = \int\left(\frac{2}{x+2} - \frac{4}{2x+1}\right)\mathrm{d}x$	Separating variables	M1
	$\ln\lvert y-1\rvert = 2\ln\lvert x+2\rvert - 2\ln\lvert 2x+1\rvert + c$		
	$\ln\lvert y-1\rvert = \ln\left\lvert\frac{(x+2)^2}{(2x+1)^2}\right\rvert + c$	Attempting to integrate, at least one term correct	M1
	$y = A\frac{(x+2)^2}{(2x+1)^2} + 1$	Attempting to rearrange into the form $y = \ldots$	M1
	$y = A\left(\frac{x+2}{2x+1}\right)^2 + 1$	Correct equation. Must include A (or equivalent).	A1
24.6 (b)	$3 = A\times\frac{3^2}{3^2} + 1 \Rightarrow A = 2$	Substituting given point	M1
	Hence $y = 2\left(\frac{x+2}{2x+1}\right)^2 + 1$	Correct result	A1

Question	Answer	Extra information	Marks
	Total		**8 marks**
24.7	$\int e^{-y}dy = \int x dx$	Separating the variables	M1
	$-e^{-y} = \frac{1}{2}x^2 + c$	Attempting to integrate	M1
	When $x = 3$, $y = \ln 4$		
	$-\frac{1}{4} = \frac{9}{2} + c$	Substituting into their result	M1
	Hence $c = -\frac{19}{4}$	Correct c	A1
	$e^{-y} = \frac{19}{4} - \frac{1}{2}x^2 \Rightarrow y = -\ln\left\|\frac{19}{4} - \frac{1}{2}x^2\right\|$	Attempting to rearrange	M1A1
	Total		**6 marks**
24.8 (a)	$\int \frac{1}{y}\, dy = \int x\cos x\, dx$	Separating the variables	M1
	$\ln y = x\sin x - \int \sin x dx$	Correct integral on LHS	A1
	$\ln y = x\sin x + \cos x + c$	Use of integration by parts on RHS	M1
	Hence $y = Ae^{x\sin x + \cos x}$	Correct integral on RHS	A1
		Correct answer. Must include A	A1
24.8 (b)	$1 = Ae^1 \Rightarrow A = e^{-1}$	Substituting	M1
	Hence $y = e^{x\sin x + \cos x - 1}$	Correct equation	A1

Question	Answer	Extra information	Marks
	Total		**7 marks**
24.9 (a)	$k = 3 \div 150$ $= 0.02$	Dividing Correct k	M1 A1
24.9 (b)	$P = 240e^{0.02t}$ $A = 240$ and $\alpha = 0.02$	e^{kt} for their k Correct answer	M1 A1
24.9 (c)	$480 = 240e^{0.02t}$ $2 = e^{0.02t}$ Hence $0.02t = \ln 2$ $t = 34.657\ldots$ $t = 35$ months	Setting their (b) = 480 Attempting to solve Correct t	M1 M1 A1
24.9 (d)	The population is predicted to increase without limit.	Any valid limitation	B1
	Total		**8 marks**
24.10 (a)	$\int \frac{1}{H-20}\,dH = -\int k\,dx$ $\ln\lvert H-20\rvert = -kx + c$ $H - 20 = e^{-kt+c}$ Hence $H = Ae^{-kt} + 20$	Separating the variables Attempting to integrate Correct expression in the form $H = \ldots$	M1 M1A1 A1
24.10 (b)	$A = 160$ $120 = 160e^{-k \times 8} + 20$ $k = 0.05875$ $k = 0.059$	 Attempting to find k with their A Correct k	B1 M1 A1

Question	Answer	Extra information	Marks
24.10 (c)	$H = 160e^{-0.059 \times 20} + 20$ $= 69.41\ldots$ $= 69.4$ °C	Substituting into their (b) Correct answer	M1 A1
	Total		**9 marks**
24.11 (a)	$\dfrac{1}{x(500-x)} = \dfrac{A}{x} + \dfrac{B}{500-x}$ $1 = A(500-x) + Bx$ Hence $A = \dfrac{1}{500}$ and $B = \dfrac{1}{500}$	Choosing correct form Substituting or equating coefficients Correct A and B	M1 M1 A1

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Question	Answer	Extra information	Marks
24.11 (b)	$\frac{1}{500}\int\left(\frac{1}{P}+\frac{1}{500-P}\right)\mathrm{d}P=\int\frac{1}{100}\mathrm{d}t$	Separating the variables and using their (a)	M1
	$\frac{1}{500}\left(\ln\lvert P\rvert-\ln\lvert 500-P\rvert\right)=\frac{1}{100}t+c$	Attempting to integrate	M1
	$\ln\left\lvert\frac{P}{500-P}\right\rvert=5t+c$	Correct equation	A1
	When $t = 0$, $P = 450$: $c=\ln 9$	Attempting to find c	M1
	Hence $\ln\left\lvert\frac{P}{500-P}\right\rvert-\ln 9=5t$		
	$\frac{P}{4500-9P}=\mathrm{e}^{5t}\Rightarrow P\mathrm{e}^{-5t}=4500-9P$		
	Hence $P=\frac{4500}{9+\mathrm{e}^{-5t}}$	Correct equation in the form $P = \ldots$	A1
24.11 (c)	No, the population will tend to 4500 ÷ 9 = 500	Correct method and reason	M1A1
	Total		**10 marks**

Question	Answer	Extra information	Marks
24.12 (a)	$x = (9-u)^2 \Rightarrow \dfrac{dx}{du} = -2(9-u)$	Attempting to find du	M1
	$\int \dfrac{1}{9-\sqrt{x}}\,dx = \int \dfrac{-2(9-u)}{u}\,du$	Attempting to substitute Correct integral in u	M1 A1
	$-\int\left(\dfrac{18}{u}-2\right)du = -18\ln u + 2u + k$	Attempting to integrate	M1
	$= 18 - 2\sqrt{x} - 18\ln\left\|9-\sqrt{x}\right\| + k$	Correct integral in x	A1
	$= -2\sqrt{x} - 18\ln\left\|9-\sqrt{x}\right\| + c$	Correct simplified answer. Must include '+ c'.	A1
24.12 (b)	$\int \dfrac{1}{9-\sqrt{x}}\,dx = \dfrac{1}{10}\int t^{\frac{1}{5}}\,dt$	Separating the variables	M1
	$-2\sqrt{x} - 18\ln\left\|9-\sqrt{x}\right\| = \dfrac{1}{12}t^{\frac{6}{5}} + c$	Attempting to integrate	M1A1
	When $t = 0$: $c = -2\sqrt{2} - 18\ln\left(9-\sqrt{2}\right)$	Attempting to find c using boundary condition	M1A1
	When $x = 8$:		
	$-2\sqrt{8} - 18\ln\left(9-\sqrt{8}\right) = \dfrac{1}{12}t^{\frac{6}{5}} - 2\sqrt{2} - 18\ln\left(9-\sqrt{2}\right)$	Attempting to substitute	M1
	Hence $t = 7.1657\ldots = 7.17$ seconds	Correct t	A1
	Total		**13 marks**

Question	Answer	Extra information	Marks
24.13 (a)	$V = 30\,000h \Rightarrow \frac{dV}{dh} = 30\,000$	Finding V and derivative	M1
	$\frac{dV}{dt} = 2000 - 120h$	Correct differential equation	B1
	Hence $\frac{dh}{dt} = \frac{2000-120h}{30\ 000}$	Using connected rate of change formula	M1
	$= \frac{50-3h}{750}$	Correct simplification	A1
24.13 (b)	$\int \frac{750}{50-3h}\,dh = \int 1\,dt$	Separating the variables	M1
	$-250\ln\|50-3h\| = t + c$	Attempting to integrate. Must include '+ c'.	M1A1
	Using $h = 30$ at $t = 0$ gives	Attempting to find c	M1
	$c = -922.219\ldots$	Correct c	A1
	When $h = 25$:		
	$t = -250\ln 25 + 922.219\ldots$	Substituting into their model	M1
	$= 117.500\ldots$		
	$= 118$ (s)	Correct time	A1
	Total		**11 marks**

Question	Answer	Extra information	Marks
24.14	$(x-3)^2+(y-4)^2=25$ $(x-3)^2+(mx-2-4)^2=25$ $x^2-6x+9+(mx)^2-12mx+36=25$ $(1+m^2)x^2-(6+12m)x+20=0$ Tangent: $b^2-4ac=0$ Hence $[-(6+12m)]^2-4\times(1+m^2)\times 20=0$ $64m^2+144m-44=0$ Hence $m=\frac{-9\pm5\sqrt{5}}{8}$	Equation of circle Substituting into their circle and attempting to expand Forming three term quadratic Use of discriminant to form three term quadratic Both solutions correct	B1 M1 M1 M1 A1
	Total		**5 marks**
24.15	$\frac{3}{2}\sin 2x=1$ $\sin 2x=\frac{2}{3}$ Hence $2x=0.729..., 2.411..., 7.012..., 8.695...$ Hence $x=0.36, 1.21, 3.51, 4.35$	Use of double angle formula for sine Use of inverse sine to find principal solution One correct value All correct values	M1 M1 A1 A1
	Total		**4 marks**