Oxford Revise | Edexcel A Level Maths | Answers

- Method (**M**) marks are awarded for showing you know a method and have attempted to apply it.
- Accuracy (**A**) marks should only be awarded if the relevant M marks have been awarded.
- Unconditional accuracy (**B**) marks are awarded independently of M marks. They do not rely on method.
- The abbreviation **o.e.** means 'or equivalent (and appropriate)'.

Please note that:

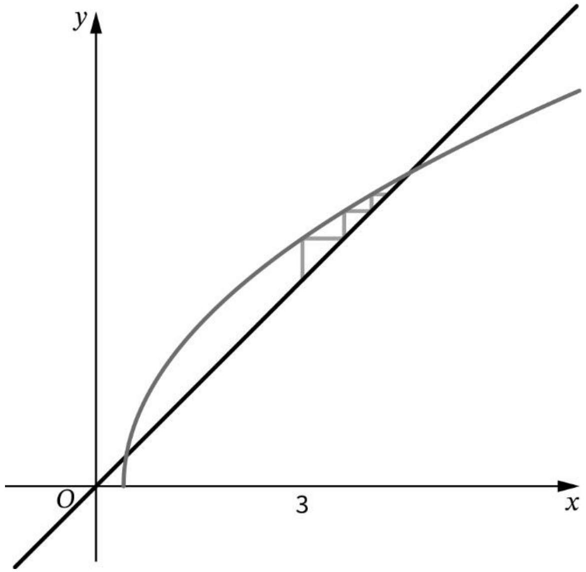
- efficient use of advanced calculators is expected
- inexact numerical answers should be given to three significant figures unless the question states otherwise; values from statistical tables should be quoted in full
- when a value of g is required, it is taken as $g = 9.8 \text{ m s}^{-2}$ unless stated otherwise in the question.

Chapter 25 Numerical methods

Question	Answer	Extra information	Marks
25.1 (a)	f(–3) = –13; f(–2) = 1 Since there is a change of sign in the interval and the function is continuous, there must be at least one root.	Evaluating function at interval end points Must state 'change of sign' and 'continuous'	M1 A1
25.1 (b)	$x^3 = 5x + 1$ $x^2 = 5 + \frac{1}{x}$ Hence $x = \pm\sqrt{5+\frac{1}{x}}$	Attempting to rearrange Must see second step for mark	M1 A1

Question	Answer	Extra information	Marks
25.1 (c)	$x_1 = -2$ $x_2 = -2.1213$ $x_3 = -2.1280$	Evidence of using correct formula, –2 is sufficient Two correct All three correct	M1 A1 A1
25.1 (d)	f(–2.1275) = 0.00788… f(–2.1285) = –0.00069… There is a change of sign so the root is correct to 3 d.p.	Use of suitable interval Must state 'change of sign'	M1 A1
	Total		**9 marks**
25.2 (a)	The change of sign is due to the presence of an asymptote.	Correct explanation	B1
25.2 (b)	f(3.2) = 3.944…; f(4) = –3 Since there is a change of sign in the interval and the function is continuous, there must be at least one root.	Evaluating function at interval end points. Must use value > 3 Must state 'change of sign' and 'continuous'	M1 A1
25.2 (c)	$5 = \frac{\sqrt{x}}{x-3} \Rightarrow x - 3 = \frac{\sqrt{x}}{5}$ Hence $x = \frac{\sqrt{x}}{5} + 3$	Attempting to rearrange Correct answer	M1 A1
25.2 (d)	$x_1 = 3.4$ $x_2 = 3.3688$ (5 s.f.) $x_3 = 3.3671$ (5 s.f.)	Evidence of using correct formula, 3.4 is sufficient Two correct All three correct	M1 A1 A1
25.2 (e)	f(3.3665) = 0.00628… f(3.3675) = –0.00659… There is a change of sign so the root is correct to 3 d.p.	Use of suitable interval Must state 'change of sign'	M1 A1

Question	Answer	Extra information	Marks
	Total		**10 marks**
25.3 (a)	f(0.5) = –0.3512…; f(0.6) = 0.1554 Since there is a change of sign in the interval and the function is continuous, there must be at least one root.	Evaluating function at interval end points Must state 'change of sign' and 'continuous'	M1 A1
25.3 (b)	$f'(x) = e^x + \frac{1}{x^2}$ Hence $x_{n+1} = x_n - \dfrac{e^{x_n} - \frac{1}{x_n}}{e^{x_n} + \frac{1}{x_n^2}}$ $x_1 = 0.5662$	Attempting to differentiate Correct derivative Attempting to use the Newton–Raphson formula with their derivative Correct approximation	M1 A1 M1 A1
25.3 (c)	$\frac{0.5671 - 0.5662}{0.5671} \times 100 = 0.1577...$ $= 0.16\%$	Use of percentage change formula Correct answer	M1 A1
	Total		**8 marks**
25.4 (a)	Where the curve and the line intersect $\sqrt{5x-2} = x$ So $5x - 2 = x^2 \Rightarrow x^2 - 5x + 2 = 0$ Setting $f(x) = x^2 - 5x + 2$ f(4) = –2; f(5) = 2 Since there is a change of sign in the interval and the function is continuous, there must be at least one root.	Attempting to form a suitable function Evaluating function at interval end points Must state 'change of sign' and 'continuous'	M1 M1 A1

Question	Answer	Extra information	Marks
25.4 (b)	 The staircase converges on the root so the iteration formula **can** be used.	Annotated graph showing staircase starting at $x = 3$ Correct answer and reason	M1 A1
25.4 (c)	Starting at $x = 1$, the iteration formula will converge to β	Correct explanation	B1
	Total		**6 marks**
25.5 (a)	f(1) = 0.8414…; f(2) = –0.0907… Since there is a change of sign in the interval and the function is continuous, there must be at least one root.	Evaluating function at interval end points Must state 'change of sign' and 'continuous'	M1 A1

Question	Answer	Extra information	Marks
25.5 (b)	$f'(x) = \cos x - 1$ Hence $x_{n+1} = x_n - \dfrac{\sin x_n - x_n + 1}{\cos x_n - 1}$ $x_1 = 1 - \dfrac{\sin 1 - 1 + 1}{\cos 1 - 1} = 2.830$ $x_2 = 2.050$	Attempting to differentiate Attempting to use the Newton–Raphson formula with their derivative One correct approximation Both approximations correct	M1A1 M1 A1 A1
25.5 (c)	The denominator of the fraction in the Newton–Raphson formula will equal zero (horizontal tangent).	Correct explanation	B1
	Total		**8 marks**
25.6 (a)	$h(3) = 14.1635\ldots$; $h(4) = -22.1285\ldots$ Since there is a change of sign in the interval and the function is continuous, there must be at least one root.	Evaluating function at interval end points Must state 'change of sign' and 'continuous'	M1 A1
25.6 (b)	$h'(t) = 30\cos t + 7\sin t - 2t$	Attempting to differentiate Correct derivative	M1 A1
25.6 (c)	$t_{n+1} = t_n - \dfrac{30\sin t_n - 7\cos t_n - t_n^2 + 12}{30\cos t_n + 7\sin t_n - 2t_n}$ $t_1 = 3 - \dfrac{30\sin 3 - 7\cos 3 - 3^2 + 12}{30\cos 3 + 7\sin 3 - 6}$ $= 3.4080$ (4 d.p.)	Use of their Newton–Raphson formula with $t_0 = 3$ Correct answer	M1 A1

Question	Answer	Extra information	Marks
25.6 (d)	$h(3.3875) = 0.01116\ldots$ $h(3.3885) = -0.02640\ldots$ There is a change of sign so the root is correct to 3 d.p.	Use of suitable interval Must state 'change of sign'	M1 A1
	Total		**8 marks**
25.7 (a)	Missing values: 0.378, 0.276 (both 3 d.p.)	Each value correct	B1B1
25.7 (b)	$A \approx \frac{1}{2} \times \frac{1}{2}\{1 + 0.212 + 2(0.561 + 0.378 + 0.276)\}$ $= 0.91$ units^2 (2 d.p.)	$h = 0.5$ Correct trapezium rule method Correct area	B1 M1A1 A1
25.7 (c)	Overestimate, because the curve is convex.	Correct conclusion and explanation	B1
25.7 (d)	$[(0.9105 - 0.88) \div 0.88] \times 100$ $= 3.47\%$	Use of percentage error formula Correct answer	M1 A1
	Total		**9 marks**
25.8 (a)	$\int x^2 e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \int \frac{2}{3} x e^{3x} dx$ $= \frac{1}{3}x^2 e^{3x} - \left[\frac{2}{9} x e^{3x} - \int \frac{2}{9} e^{3x} dx\right]$ $= \frac{1}{3}x^2 e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + c$	Use of integration by parts once Use of integration by parts twice Final integration	M1A1 M1A1 M1A1

Question	Answer	Extra information	Marks
25.8 (b)	$\left[\frac{1}{3}x^2e^{3x}-\frac{2}{9}xe^{3x}+\frac{2}{27}e^{3x}\right]_0^{\ln 2}$	Attempting to integrate their (a) and evaluating using limits given	M1
	$=\left[\frac{1}{3}(\ln 2)^2(8)-\frac{2}{9}(\ln 2)(8)+\frac{2}{27}(8)\right]-\frac{2}{27}$		
	$=\frac{14}{27}+\frac{8}{3}(\ln 2)^2-\frac{16}{9}\ln 2$	Correct answer	A1
	Total		**8 marks**
25.9	$\cos^4 x=(\cos^2 x)^2$	Writing $\cos^4 x$ in terms of $\cos^2$ and use of double angle formula	M1
	$=\left[\frac{1}{2}(1+\cos 2x)\right]^2$		
	$=\frac{1}{4}(1+2\cos 2x+\cos^2 2x)$	Attempting to expand	M1
	$=\frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\left[\frac{1}{2}(1+\cos 4x)\right]$	Use of double angle formula again and attempting to expand	M1
	$=\frac{3}{8}+\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x$	Correct expression	A1
	Total		**4 marks**